AP CALCULUS SUMMER PACKET

A: Basic Algebra Skills

A1. True or false. If false, change what is underlined to make the statement true.

a. $(x^3)^4 = x^{\underline{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\frac{3}{2}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3} + 2\sqrt{x-3} = 5\sqrt{x-3}$ T F

g. If $(x+3)(x-10) = \underline{2}$, then $x+3 = \underline{2}$ or $x-10 = \underline{2}$. T F

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T: Trigonometry

You should be able to answer these quickly, *without* using calculator and without referring to (or drawing) a unit circle.

T1. Evaluate Trig Functions without a calculator:

1. $\cos\pi$
2. $\sin\dfrac{\pi}{6}$
3. $\sec 210^\circ$
4. $\tan 90^\circ$
5. $\csc(-150)$
6. $\csc\dfrac{3\pi}{2}$
7. $\cos 0$
8. $\sin^{-1}\dfrac{-1}{2}$
9. $\mathrm{Cos}^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$
10. $\tan^{-1}1$
11. $\arcsin 0$
12. $\mathrm{Tan}^{-1}\left(-\sqrt{3}\right)$
13. $\sin\dfrac{2\pi}{3}$
14. $\mathrm{Sin}^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$
15. $\arctan 0$

T2. Find the value of each expression, in exact form.

a. $\sin\frac{2\pi}{3}$ **b.** $\cos\frac{11\pi}{6}$

c. $\tan\frac{3\pi}{4}$ **d.** $\sec\frac{5\pi}{3}$

e. $\csc\frac{7\pi}{4}$ **f.** $\cot\frac{5\pi}{6}$

Note: You will need to know your trig identities, Sum & Difference & Double Angle Formulas:

Memorize the following Trig Identities:

$\sin^2\theta+\cos^2\theta=1$ $\quad$ $\sec^2\theta=1+\tan^2\theta$ $\quad$ $\csc^2\theta=1+\cot^2\theta$

$\sin(-\theta)=-\sin\theta$ $\quad$ $\cos(-\theta)=\cos\theta$ $\quad$ $\tan(-\theta)=-\tan\theta$

$\cos^2\theta=\dfrac{1+\cos 2\theta}{2}$ $\quad$ $\sin^2\theta=\dfrac{1-\cos 2\theta}{2}$

$\sin 2\theta=2\sin\theta\cos\theta$ $\quad$ $\cos 2\theta=\cos^2\theta-\sin^2\theta$

T3 Find the value(s) of x in $[0, 2\pi)$ which solve each equation.

a. $\sin x=\frac{\sqrt{3}}{2}$ **b.** $\cos x=-1$

c. $\tan x=\sqrt{3}$ **d.** $\sec x=-2$

e. $\csc x$ is undefined **f.** $\cot x=1$

T4. Solve the equation. Give *all* real solutions, if any.

a. $\sin 3x=1$ **b.** $2\sqrt{3}\cos(\pi x)=3$

c. $\tan 2x=0$ **d.** $4\sec x+1=9$

e. $\csc(4x+3)=0$ **f.** $3\cot 6x+\sqrt{3}=0$

T5. Solve by factoring. Give *all* real solutions, if any.

a. $4\sin^2 x+4\sin x+1=0$ **b.** $\cos^2 x-\cos x=0$

c. $\sin x\cos x-\sin^2 x=0$ **d.** $x\tan x+3\tan x=x+3$

T6. Graph each function, identifying x- and y-intercepts, if any, and asymptotes, if any.

a. $y=-\sin(2x)$ **b.** $y=4+\cos x$

c. $y=\tan x-1$ **d.** $y=\sec x+1$

e. $y=\csc(\pi x)$ **f.** $y=2\cot x$

S: Solving

S1. Solve by factoring.

a. $x^3 + 5x^2 - x - 5 = 0$

b. $4x^4 + 36 = 40x^2$

c. $(x^3 - 6)^2 + 3(x^3 - 6) - 10 = 0$

d. $x^5 + 8 = x^3 + 8x^2$

S2. Solve by factoring. You should be able to solve each of these *without* multiplying the whole thing out. (In fact, for goodness' sake, please *don't* multiply it all out!)

a. $(x + 2)^2 (x + 6)^3 + (x + 2)(x + 6)^4 = 0$

b. $(2x - 3)^3 (x^2 - 9)^2 + (2x - 3)^5 (x^2 - 9) = 0$

c. $(3x + 11)^5 (x + 5)^2 (2x - 1)^3 + (3x + 11)^4 (x + 5)^4 (2x - 1)^3 = 0$

d. $6x^2 - 5x - 4 = (2x + 1)^2 (3x - 4)^2$

S3. Solve. (*Hint*: Each question *can* be solved by factoring, but there are other methods, too)

a. $a(3a + 2)^{1/2} + 2(3a + 2)^{3/2} = 0$

b. $\sqrt{2x^2 + x - 6} + \sqrt{2x - 3} = 0$

c. $2\sqrt{x + 3} = x + 3$

d. $\dfrac{6}{(2x + 1)^2} + \dfrac{3}{2x + 1} = 1 + \dfrac{2}{2x + 1}$

S4. Solving Inequalities: *Solve and graph the solution*

a. $|x - 3| > 12$ **b.** $|x - 3| \le 4$ **c.** $|10x + 8| > 2$

d. $x^2 - 16 < 0$ **e.** $x^2 + 6x - 16 \le 0$ **f.** $x^2 - 3x \ge 10$

L: Logarithms and Exponential Functions

L1. Evaluate Logarithms and Exponentials without a calculator

a. $\log_4 64$ **b.** $\log_3 \frac{1}{9}$ **c.** $\log 10$ **d.** $\ln e$

e. $\ln 1$ **f.** $\ln e^3$ **g.** $3^{\log_3 7}$ **h.** $4^{\log_4 \sin x}$

L2. Expand as much as possible.

a. $\ln x^2 y^3$ **b.** $\ln \frac{x+3}{4y}$

c. $\ln 3\sqrt{x}$ **d.** $\ln 4xy$

L3. Condense into the logarithm of a single expression.

a. $4\ln x + 5\ln y$ **b.** $\frac{2}{3}\ln a + 5\ln 2$

c. $\ln x - \ln 2$ **d.** $\frac{\ln x}{\ln 2}$
(contrast with part **c**)

L4. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $\ln (x+3) = 2$ **b.** $\ln x + \ln 4 = 1$

c. $\ln x + \ln (x+2) = \ln 3$ **d.** $\ln (x+1) - \ln (2x-3) = \ln 2$

L5. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $e^{4x+5} = 1$ **b.** $2^x = 8^{4x-1}$

c. $100e^{x \ln 4} = 50$ **d.** $2^x = 3^{x-1}$
(need rounded answer only in **d**)

L6. Round final answers to 3 decimal places.

a. At $t = 0$ there were 140 million bacteria cells in a petri dish. After 6 hours, there were 320 million cells. If the population grew exponentially for $t \geq 0$...

...how many cells were in the dish 11 hours after the experiment began?

...after how many hours will there be 1 billion cells?

b. The *half-life* of a substance is the time it takes for half of the substance to decay. The *half-life* of Carbon-14 is 5568 years. If the decay is exponential...

...what percentage of a Carbon-14 specimen decays in 100 years?

...how many years does it take for 90% of a Carbon-14 specimen to decay?

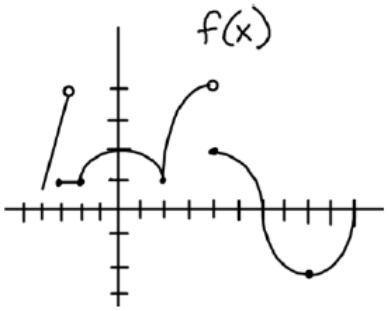
F: FUNCTIONS

Graph each of the following Parent Functions and be familiar with these graphs

1. $f(x) = x$
2. $f(x) = x^2$
3. $f(x) = x^3$
4. $f(x) = |x|$
5. $f(x) = \sqrt{x}$
6. $f(x) = \frac{1}{x}$
7. $f(x) = \frac{1}{x^2}$
8. $f(x) = e^x$
9. $f(x) = \ln x$
10. $f(x) = \sin x$
11. $f(x) = \cos\ x$
12. $f(x) = \tan x$
13. $f(x) = \tan^{-1} x$
14. $f(x) = x^{\frac{2}{3}}$
15. $f(x) = \frac{1}{1+x^2}$
16. $f(x) = [x]$
17. $f(x) = \sqrt{1-x^2}$
18. $f(x) = \frac{|x|}{x}$

Analyzing Functions

F1. Increasing/Decreasing



Determine the interval(s) over which f(x) is:

a. Increasing _____

b. Decreasing _____

c. Constant _____

d. Linear _____

e. Concave Up _____

f. What are the zeros of f? _____

g. For what values of x is f(x) discontinuous? _____

F2. Compositions

1. Let $f(x) = 3x^2$ and $g(x) = \frac{x-9}{x+1}$, find the following:

   a. $f(g(x))$ b. $g(f(x))$ c. $f^{-1}(x)$

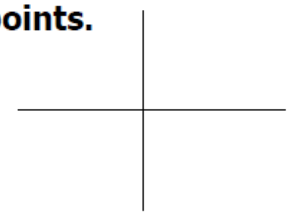
   d. Domain, Range, and Zeros of f(x) e. Domain, Range, and Zeros of g(x)

Find f^{-1} and verify that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

2. $f(x) = 2x + 3$
3. $f(x) = x^3 - 1$

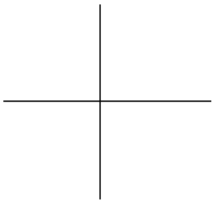
F3. Piecewise Functions:
Graph and then evaluate the function at the indicated points.

1. $f(x)=\begin{cases} 3x+2, & x>3 \\ -x+4, & x\le 3 \end{cases}$



a. f(2) b. f(3) c. f(5)

2. $f(x)=\begin{cases} x^2\text{-}1, & x<\text{-}2 \\ 4, & \text{-}2\le x\le 1 \\ 3x+1, & 1<x<3 \\ x^2-1, & x>3 \end{cases}$



a. f(-3) b. f(-2) c. f(2) d. f(5) e) f(3)

F4. Even/Odd Functions

Show work to determine if the relation is even, odd, or neither.

a. $f(x)=2x^2-7$ **b.** $f(x)=-4x^3-2x$ **c.** $f(x)=4x^2-4x+4$

d. $f(x)=x-\frac{1}{x}$ **e.** $f(x)=|x|-x^2+1$ **f.** $f(x)=\sin x+x$

F5. Domains of Functions: Find the Domain of each.

a. $y=\frac{3x-2}{4x+1}$ **b.** $y=\frac{x^2-4}{2x+4}$ **c.** $y=\frac{x^2-5x-6}{x^2-3x-18}$

d. $y=\frac{2^{2-x}}{x}$ **e.** $y=\sqrt{x-3}-\sqrt{x+3}$ **f.** $y=\frac{\sqrt{2x-9}}{2x+9}$

F6. Asymptotes

Find the equation of both Horizontal and Vertical Asymptotes for the following functions. Find the coordinates of any holes.

a. $y=\frac{x}{x-3}$ **b.** $y=\frac{x+4}{x^2-1}$

c. $y=\frac{x^2-2x+1}{x^2-3x-4}$ **d.** $y=\frac{x^2-9}{x^3-3x^2-18x}$

R: Rational Expressions and Equations

R1.

	Function	Domain	Hole(s): (x, y) if any	Horiz. Asym., if any	Vert. Asym.(s), if any
a.	$f(x) = \dfrac{4x^2 + 7x - 15}{8x^2 - 14x + 5}$				
b.	$f(x) = \dfrac{3(4 + x)^2 - 48}{x}$				
c.	$f(x) = \dfrac{6x + 4}{\sqrt{3x^2 - 10x - 8}}$		skip	skip	

R2. Write the equation of a function that has...

a. asymptotes $y = 4$ and $x = 1$, and a hole at (3, 5)

b. holes at (-2, 1) and (2, -1), an asymptote $x = 0$, and no horizontal asymptote

R3. Find the x-coordinates where the function's output is zero and where it is undefined.

a. For what real value(s) of x, if any, is the output of the function $f(x) = \dfrac{x^2 + 4}{e^{6x} - 1}$

...equal to zero? ...undefined?

b. For what real value(s) of x, if any, is the output of $g(x) = \dfrac{\cos^2(\pi x)}{\sin x + 2}$...

...equal to zero? ...undefined?

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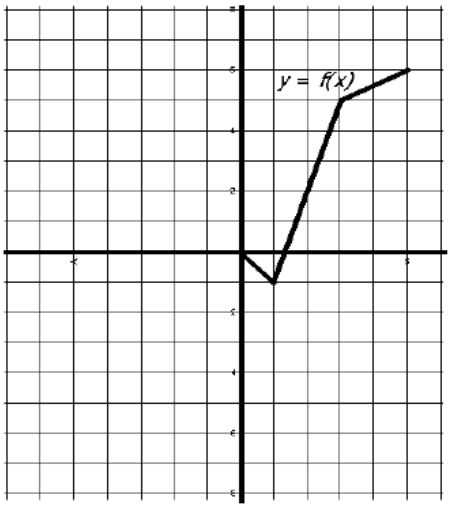
G: Graphing

G1. PART of the graph of f is given. Each gridline represents 1 unit.



a. Complete the graph to make f an EVEN function.

b. What are the domain and range of f_{even}?

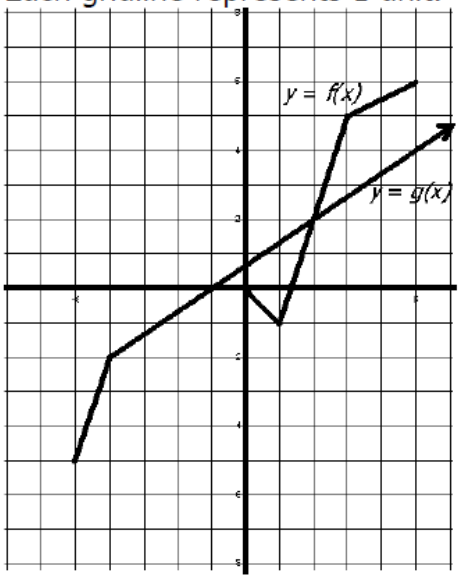
c. What is $f_{even}(-3)$?

d. Complete the graph to make f an ODD function.

e. What are the domain and range of f_{odd}?

f. What is $f_{odd}(-3)$?

G2. The graphs of f and g are given. Answer each question, if possible. If impossible, explain why. Each gridline represents 1 unit.



a. $f^{-1}(5)$ =

b. $f(g(5))$ =

c. $(g \circ f)(3)$ =

d. Solve for x: $f(g(x)) = 5$

e. Solve for x: $f(x) = g(x)$

For parts **f – i**, respond in interval notation.

f. For what values of x is $f(x)$ increasing?

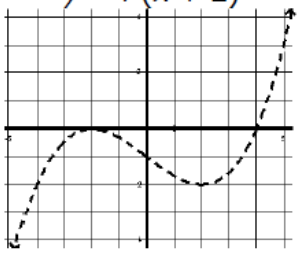
g. For what values of x is $g(x)$ positive?

h. Solve for x: $f(x) < 4$

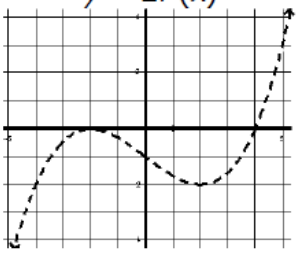
i. Solve for x: $f(x) \geq g(x)$

G3. Given the graph of $y = f(x)$ (dashed graph), sketch each transformed graph.

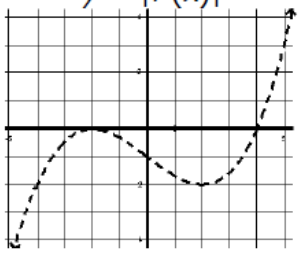
a. $y = f(x + 2)$



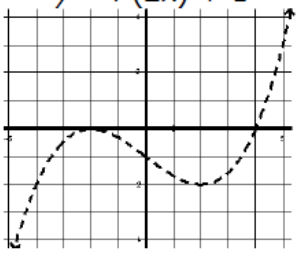
b. $y = 2f(x)$



c. $y = |f(x)|$



d. $y = f(2x) + 1$



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PO: Polar Functions

PO1. Plot the point $\left(3, -\frac{3\pi}{4}\right)$ and find three additional represents of this point using $-2\pi < \theta < 2\pi$.

PO2. Convert the given points in polar into rectangular coordinates

(a) $\left(\sqrt{3}, \frac{\pi}{6}\right)$, (b) $\left(2, \frac{2\pi}{3}\right)$, (c) $\left(-3, \frac{-3\pi}{4}\right)$ (d) $\left(-2, \frac{5\pi}{6}\right)$.

PO3. Convert the given points in rectangular into polar coordinates,

(a) (0, 2), (b) $\left(-1, \sqrt{3}\right)$, (c) $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ (d) $\left(\sqrt{3}, -1\right)$.

PO4. Convert the following polar equations into rectangular form:

a. r=2 **b.** $\theta = \frac{\pi}{3}$ **c.** $r = \sec\theta$ **d.** $r = 3\cos\theta + 2\sin\theta$

PO5. Convert the following rectangular equations into polar form

a. y = x **b.** x= 10 **c.** $x^2+y^2=4$ **d.** $x^2-y^2=4x$

PO6. Sketch the graph of the following polar equation:

$r=3+2\cos\theta$

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PA: Parametric Functions

Obtain the rectangular equation by eliminating the parameter. Sketch a graph using the parametric equations:

(a) $x=2t-5$, $y=4t-7$ (b) $x=4-\sqrt{t}$, $y=\sqrt{t}$ (c) $x=t^2$, $y=\sqrt{4-t^2}$

(d) $x=4\cos\theta$, $y=2\sin\theta$ (e) $x=9\sin^2\theta$, $y=9\cos^2\theta$ (f) $x=\sec^2\theta-1$, $y=\tan\theta$

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PF: Partial Fractions

PF1. Find the partial fraction decomposition of

(a) $\dfrac{2x-1}{(x-2)(x-3)}$ (b) $\dfrac{x+7}{x^2-x-6}$ (c) $\dfrac{x^2+2}{(x-1)(x+2)(x-3)}$